Abstract: In this paper, the multisensor systems with ARMA colored measurement noise are converted to those with the same local dynamic model and uncorrelated noises by using the state augmented method. Furthermore, the globally optimal weighted measurement fusion Kalman filter is given. Compared with the existing methods transforming the multisensor systems into those with the same or different local dynamic models and correlated noises, the transformed system models in this paper are applicable to the weighted measurement fusion algorithm. Although the dimension of augmented state is higher than that in the existing references, the computational load and complexity of the whole optimal fusion Kalman filter are better than the existing fusers. The simulation example for a 3-sensors radar tracking system with colored measurement noises shows its effectiveness.

Key words: Multisensor information fusion; weighted measurement fusion; Kalman filter; colored measurement noises; radar tracking system

I. INTRODUCTION

Recently, multisensor information fusion Kalman filtering has been widely applied in many fields such as military, defense, target tracking, guidance, and so on. For Kalman filtering-based fusion, two basic fusion methods are state and measurement fusion methods (Gao et al., 2002; Chen et al., 2007). The state fusion method includes the centralized and weighted fusion methods. The centralized state fusion method can give the globally optimal state estimation by directly combining the local measurement data, but its disadvantages is that it may require a larger computational burden. The weighted state fusion method can give the globally suboptimal state estimation by weighting the local state estimators. This method has considerable advantages that it can facilitate fault detection and isolation more conveniently, and can reduce the computational burden. The measurement fusion method also includes centralized and weighted fusion methods. The centralized measurement fusion method is identical to the centralized state fusion method. The weighted measurement fusion method weights the local measurements to obtain a fused measurement equation, and then uses a single Kalman filter to obtain the final fused state estimation. It is functional equivalent to the centralized fusion Kalman filter under certain conditions (Gan et al., 2001), so that it also has global optimality.

The research on the state estimation for the systems with colored noise has never stopped (Gao et al., 2011; Gan et al., 2011). Gao (2011) and Gan (2011) presented the Kalman filters for the single sensor system with colored noises. Now, there are mainly two kinds of fusion state estimation methods for the multisensor systems with colored measurement noises: one is to convert the multisensor systems with colored measurement noises to those with the same local dynamic model and correlated noises (Sun et al., 2004; Sun et al., 2005). But there has not been a kind of weighted measurement fusion algorithm applicable to these systems. The global suboptimal weighted state fusion Kalman filters weighted by scalars and matrices are respectively given by Sun (2004) and Sun (2005). The other is to convert the multisensor systems with colored measurement noises to those with different local dynamic model, which are also not the suitable systems for the weighted measurement fusion methods. Three weighted state fusion Kalman filters are given based on the Kalman filtering method and modern time series analysis method, respectively (Deng et al., 2005; Sun et al., 2007). In a word, the existing methods of state estimation for the multisensor systems with colored measurement noises all give the global suboptimal Kalman fusers, and do not solve the global optimal state estimation problem. In this paper, the multisensor systems with colored measurement noises are converted to those with the same local dynamic model and independent noises by the state augmented method different from that in the existing references. Furthermore, the global optimal weighted fusion Kalman filter is given based on the weighted measurement fusion algorithm.
II. PROBLEM FORMULATION

Consider the multisensor discrete stochastic control system with colored measurement noises

\[ x(t+1) = \Phi x(t) + Bu(t) + \Gamma w(t) \]  
(1)

\[ y_i(t) = \tilde{H}_i x(t) + \eta_i(t) + v_i(t), \quad i = 1, 2, \ldots, L \]  
(2)

\[ P_i(q^{-1}) \eta_i(t) = R_i(q^{-1}) \xi_i(t) \]  
(3)

where \( t \) is the discrete time, \( x(t) \in \mathbb{R}^n \) is the state, \( y_i(t) \in \mathbb{R}^{m_i} \) is the measurement of \( i \)th sensor subsystem, \( u(t) \in \mathbb{R}^m \) is the control, \( w(t) \in \mathbb{R}^r \) is the input noise, \( \eta_i(t) \in \mathbb{R}^{p_i} \) and \( v_i(t) \in \mathbb{R}^{q_i} \) are the measurement noises, \( w(t) \), \( v_i(t) \) and \( \xi_i(t) \) are the independent white noises with zero means and variance matrices \( Q_w \), \( Q_{vi} \) and \( Q_{\xi_i} \). \( \eta_i(t) \) is the colored measurement noise obeying the ARMA model in (3), \( \Phi \), \( B \), \( \Gamma \) and \( H \) are constant matrices with compatible dimensions, \( P_i(q^{-1}) = 1 + p_{i1}q^{-1} + \cdots + p_{i,m_i}q^{-m_i} \), and \( R_i(q^{-1}) = r_{i1}q^{-1} + \cdots + r_{i,m_i}q^{-m_i} \).

**Assumption 1** The initial value \( x(0) \) is independent of \( w(t) \), \( \eta_i(t) \) and \( v_i(t) \) (\( i = 1, \ldots, L \)), and

\[ \mathbb{E}[x(0)] = \mu_0, \quad \mathbb{E}[(x(0) - \mu_0)(x(0) - \mu_0)^T] = P_0 \]  
(4)

**Assumption 2** The control \( u(t) \) is known input.

**Assumption 3** \((\Phi, \Gamma)\) is a completely observable pair, and \((\Phi, \Gamma)\) is a completely controllable pair.

The problem is to find the local steady-state Kalman filter \( \hat{x}_i(t | t) \) and globally optimal weighted measurement fusion Kalman filter \( \hat{x}(t | t) \) for the state \( x(t) \).

III. AUGMENTED TRANSFORMATION FOR STATE SPACE MODEL

As given by Deng (2005; 2004), Sun (2007) and Yu (2006), the system (1)-(3) is converted to a multisensor system with different local dynamic models using the augmented state space method, and the weighted measurement fusion algorithm cannot be used to yield the globally optimal state fuser. In this paper, the multisensor system (1)-(3) is converted to that with the same local dynamic model and uncorrelated noises using the method different from these given by Deng (2005; 2004), Sun (2007) and Yu (2006).

From (3), we have the state space model for the colored measurement noise \( \eta_i(t) \) as follows

\[ \alpha_i(t+1) = P_i \alpha_i(t) + R_i \xi_i(t) \]  
(5)

\[ \eta_i(t) = H_i \alpha_i(t) \]  
(6)

\[ P_i = \begin{bmatrix} -p_{i1} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ -p_{i,m_i} & \ldots & 0 \end{bmatrix}, \quad R_i = \begin{bmatrix} r_{i1} \\ \vdots \\ r_{i,m_i} \end{bmatrix} \]

\[ H_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \]  
(7)

Then we have the augmented state space model with the same local dynamic model

\[ x_i(t+1) = \Phi_i x_i(t) + \Gamma_i w_i(t) \]  
(8)

\[ y_i(t) = H_{ni} x_i(t) + v_i(t) \]  
(9)

\[ x(t) = C_i x_i(t) \]  
(10)

with the definitions

\[ x_i(t) = \begin{bmatrix} x(t) \\ \alpha_i(t) \\ \vdots \\ \alpha_L(t) \\ w_i(t) \end{bmatrix}, \quad w_i(t) = \begin{bmatrix} \xi_i(t) \\ \vdots \\ \xi_L(t) \end{bmatrix} \]  
(11)

\[ \Phi_i = \begin{bmatrix} \Phi & 0 & \cdots & 0 \\ 0 & P_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_L \end{bmatrix}, \quad \Gamma_i = \begin{bmatrix} \Gamma & 0 & \cdots & 0 \\ 0 & R_i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_L \end{bmatrix} \]

\[ H_n = \begin{bmatrix} \tilde{H}_i & 0 & \cdots & H_i & \cdots & 0 \end{bmatrix}, \quad C_i = \begin{bmatrix} I_n & 0 & \cdots & 0 \end{bmatrix} \]  
(12)

IV. GLOBALLY OPTIMAL WEIGHTED MEASUREMENT FUSION KALMAN FILTER

For the multisensor system (1)-(3), the centralized measurement fusion equation is given as

\[ y^{(0)}(t) = H^{(0)} x_i(t) + v^{(0)}(t) \]  
(13)

with the definitions

\[ y^{(0)}(t) = \begin{bmatrix} y_1^T(t) \\ \vdots \\ y_L^T(t) \end{bmatrix} \]  
(14)

\[ H^{(0)} = \begin{bmatrix} H_{n1}^T, \ldots, H_{nL}^T \end{bmatrix} \]  
(15)

\[ v^{(0)}(t) = \begin{bmatrix} v_1^T(t) \\ \vdots \\ v_L^T(t) \end{bmatrix} \]  
(16)

and the fused measurement white noise \( v^{(0)}(t) \) has the variance matrix \( R^{(0)} \) as follows
\[ R^{(0)} = \text{block-diag}(R_s, \ldots, R_c) \]  
where \( R_i = Q_{i\pi} \).

For the system (8) and (13), the centralized fusion Kalman filter \( \hat{x}^{(0)}(t \mid t) \) and filtering error variance matrix \( P^{(0)} \) can be obtained using the standard Kalman filtering algorithm (Deng, 2004).

Assume that \( H_{\pi} \) has the common right factor matrix \( H_r \) in \( m \times n \) dimension, i.e.
\[ H_{\pi} = M_i H_r, \quad i = 1, \ldots, L \]  
and suppose that the following inverse matrix exists
\[ [M^{(0)} R^{(0)-1} M^{(0)}]^{-1} \]  
where
\[ M^{(0)} = [M_1^T, \ldots, M_L^T]^T \]  
then we have the centralized measurement fusion equation as follows
\[ y^{(0)}(t) = M^{(0)} H_r x(t) + v^{(0)}(t) \]  
Eq. (21) can be considered as the measurement equation for \( H_r x(t) \), and the weighted least square (Gauss-Markov) estimator for \( H_r x(t) \) is given by
\[ y_r(t) = [M^{(0)T} R^{(0)-1} M^{(0)}]^{-1} M^{(0)T} R^{(0)-1} y^{(0)}(t) \]

Then we have the following weighted measurement fusion model
\[ y_r(t) = H_r x_r(t) + v_r(t) \]  
where \( v_r(t) \) is the measurement error, and the least measurement error variance matrix is given by
\[ R = [M^{(0)T} R^{(0)-1} M^{(0)}]^{-1} \]  
Substituting (21) into (22) yields (23) and \( v_r(t) \),
\[ v_r(t) = [M^{(0)T} R^{(0)-1} M^{(0)}]^{-1} M^{(0)T} R^{(0)-1} v^{(0)}(t) \]  
Obviously, \( v_r(t) \) is a white noise. For the system (8) and (23), the weighted measurement fusion Kalman filter \( \hat{x}_r(t \mid t) \) and filtering error variance matrix \( P_r \) can be obtained using the standard Kalman filtering algorithm (Deng, 2004).

**Theorem 1** For the multisensor system (8) and (23) with Assumptions 1-3, the weighted measurement fusion steady-state optimal Kalman filter and predictor are given by
\[ \hat{x}(t \mid t-1) = C_r \hat{x}_r(t \mid t-1) \]  
\[ \hat{x}(t \mid t) = C_r \hat{x}_r(t \mid t) \]  
\[ \Sigma = C_r \Sigma_r C_r^T \]  
\[ P = C_r P_r C_r^T \]  
\[ \hat{x}_r(t \mid t-1) = \Phi_r \hat{x}_r(t-1 \mid t-1) \]  
\[ \Sigma_r = \Phi_r P_r \Phi_r^T + \Gamma_r Q_r \Gamma_r^T \]  
\[ K_r = \Sigma_r H_r^T (R_r)^{-1} \]  
\[ R_r = H_r \Sigma_r H_r^T + R \]  
where \( P \) and \( P_r \) are the steady-state filtering error variance matrices, and \( \Sigma \) and \( \Sigma_r \) are the steady-state prediction error variance matrices.

**Proof.** For the multisensor system (8) and (23), we can obtain the globally optimal weighted measurement fusion Kalman filter (30)-(36), using the standard Kalman filtering method (Deng, 2004). Applying (10) and the projective property yields (26) and (27). Subtracting (26) from (10) yields \( \hat{x}(t \mid t-1) = C_r \hat{x}_r(t \mid t-1) \) , which yields (28). Subtracting (27) from (10) yields \( \hat{x}(t \mid t) = C_r \hat{x}_r(t \mid t) \) , which yields (29). The proof is completed.

**V. COMPARATIVE ANALYSIS ON THE COMPUTATIONAL LOCAL AND PERFORMANCE**

From (1), the state \( x(t) \) is of \( n \) dimension, and \( \alpha_i(t) \) is of \( n_{\pi_i} \) dimension shown by (5). As given by Sun (2004; 2005), \( L \) local Kalman filters in \( n \) dimensions and \( L^2 \) local estimation error variance and covariance matrices in \( n \) dimensions require to be computed in order to compute the weights of the weighted state fusion Kalman filter. As given by Deng (2005; 2004), Sun (2007) and Yu (2006), \( L \) local Kalman filters in \( n + n_{\pi_i} \) dimensions and \( L^2 \) local estimation error variance and covariance matrices in \( n + n_{\pi_i} \) dimensions required to be computed in order to compute the weights of the weighted state fusion Kalman filter. In this paper, the local Kalman filter need not to be computed, and only a fused Kalman filter in \( n + L n_{\pi_i} \) dimension requires to be computed after weighting local measurements. It is shown that the whole fused Kalman filter is better than these given
VI. SIMULATION EXAMPLE

Consider the 3-sensors tracking system with colored measurement noises

$$x(t+1) = \mathbf{F}x(t) + \mathbf{G}_0w(t)$$

$$y_i(t) = H_\alpha x(t) + \eta_i(t) + v_i(t)$$

where $\mathbf{F} = \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{G}_0 = \begin{bmatrix} 0.5T_0^2 \\ T_0 \end{bmatrix}$, $H_\alpha = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (37)

$$\mathbf{F}_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & P_1 & 0 \\ 0 & 0 & P_2 \end{bmatrix}$$,

$$\mathbf{G}_r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}$$

$$p_{11} = -0.8, \quad p_{12} = 0.3, \quad p_{21} = -0.7, \quad p_{22} = 0.4, \quad p_{31} = -0.6, \quad p_{32} = 0.5$$ (41)

In simulation, we take $T_0 = 0.4, \quad \sigma_{\eta_1}^2 = 1, \quad \sigma_{\eta_2}^2 = 1, \quad \sigma_{\eta_3}^2 = 4, \quad \sigma_{\eta_4}^2 = 8, \quad \sigma_{\eta_5}^2 = 4, \quad \sigma_{\eta_6}^2 = 8, \quad \sigma_{\eta_7}^2 = 3$, and we define

$$\mathbf{C}_r = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From (48), we have the common right factor for each measurement matrix $H_\alpha$ as follows

$$H_\alpha = \begin{bmatrix} H_{\alpha_0} & H_{\alpha_1} & 0 & 0 \\ 0 & 0 & H_{\alpha_1} & 0 \\ 0 & 0 & 0 & H_{\alpha_1} \end{bmatrix}$$

and

$$M_i = [1 \ 0 \ 0], \quad M_j = [0 \ 1 \ 0], \quad M_k = [0 \ 0 \ 1]$$

As given by Deng (2004), the theoretical accuracy of the weighted state fusion Kalman filter weighted by scalars is 2.6738, that of the weighted state fusion Kalman filter weighted by diagonal matrices is 2.6731, that of the weighted state fusion Kalman filter weighted by matrices is 2.6705, and that of the weighted measurement fusion Kalman filter in this paper is 2.3804. As given by Deng (2004), the theoretical accuracy of the weighted state fusion Kalman predictor weighted by scalars is 3.6663, that of the weighted state fusion Kalman predictor weighted by diagonal matrices is 3.6657, that of the weighted state fusion Kalman predictor weighted by matrices is 3.6624, and that of the weighted measurement fusion Kalman predictor in this paper is 3.2640. These show that the weighted measurement fusion Kalman filter and predictor given in this paper improve the state estimation accuracy for the multisensor systems with colored measurement noises.
VII. CONCLUSIONS

The multisensor system with ARMA colored measurement noises is converted to that with the same local dynamic model and uncorrelated noises by using the state augmented method. Furthermore, the globally optimal weighted measurement fusion Kalman filter is given by using the weighted measurement fusion algorithm. The transformation method in this paper is better than that in the existing references, which gives the globally optimal weighted measurement fusion Kalman filter. Moreover, the simple analysis on the computational load and complex performance for the proposed Kalman fuser is also given compared with the weighted state fusion Kalman fusers in the existing reference.

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IX. REFERENCES


