Analysis of Novel Eigen Beam Forming Scheme with Power Allocation in LSAS

Saransh Malik, Sangmi Moon, Hun Choi, Cheolhong Kim, Daejin Kim, and Intae Hwang,

Non-Member, IEEE

Abstract—Massive MIMO (also known as “Large-Scale Antenna Systems”, “Very Large MIMO”, “Hyper MIMO”, “Full-Dimension MIMO” and “ARGOS”) enables a significant reduction of latency on the air interface with the use of a large excess of service-antennas over active terminals and time division duplex operation. We consider the downlink of a time-division duplexing (TDD) multiccell multuser MIMO system where the base stations (BSs) are equipped with a very large number of antennas. Assuming channel estimation through uplink pilots, arbitrary antenna correlation and user distributions, we derive approximations of achievable rates with linear precoding techniques, namely Zero Forcing (ZF), Matched Filtering (MF), Eigen-Beam Forming (EBF) and Regularized Zero-Forcing (RZF). The approximations are tight in the large system limit with an infinitely large number of antennas and user terminals (UTs), but match our simulations for realistic system dimensions. We further show that a simple EBF precoding scheme can achieve the same performance as RZF with one order of magnitude fewer antennas in both uncorrelated and correlated fading channels. Our simulation results show that our proposal is a better precoding scheme than the conventional scheme. Also, we have used two channel environment channels for further analysis of our algorithm given as Long Term Evolution Advanced (LTE-A) and another one is Millimeter wave Mobile Broadband (MBB) channel.

Index Terms—Eigen Beam Forming (EBF), LSAS, LTE-A, MF, MMB, Power Allocation, RZF, ZF.

I. INTRODUCTION

USE of multiple antennas at the base stations (BSs) is an integral part of future wireless cellular systems [1-3] as it allows serving multiple User Terminals (UTs) simultaneously on the same resource block and to counter inter and intracell interference [4]. However, these advantages come at the cost of overhead for the acquisition of channel state information (CSI) at the BSs. In Frequency-Division Duplexing (FDD) systems, this overhead scales linearly with the number of antennas and renders the use of very large antenna arrays essentially impossible [5]. In Time-Division Duplexing (TDD) systems where channel reciprocity can be exploited, the training overhead scales linearly with the number of UTs. Hence, additional antenna elements can be added at no overhead cost to significantly improve the system performance [6]. In Fig. 1, we show the exemplary LSAS system proposed in recent researches [7]. The simplest forms of precoding and detection, i.e., beam forming (BF) and matched filtering (MF), become optimal and the transmit power at BSs and UTs can be made arbitrarily small. It has been proved from previous work that the system performance is limited by pilot contamination, the simplest precoder or detector, i.e. Maximum Ratio Transmission (MRT) precoding and Matched Filter (MF), are optimal, and the transmission power can be made arbitrarily small when the number of antennas approaches infinite [7,8]. Numerous papers have researched on transmission mechanisms of MU-MIMO, Simple Zero-Forcing (ZF) based linear algorithms were proposed in [9] and [10] for MU-MIMO where the transmitters and receivers are equipped with multiple antennas. In a single-cell system, it is always advantageous to have an unlimited number of antennas at the transmitter [1] and also at the receiver. In [11], the author proposed massive MIMO systems using a simple linear algorithm such as maximal ratio transmission (MRT) in downlink and maximal ratio combining (MRC) in uplink. In [12], the downlink performance of MRT and ZF Beam Forming for massive MIMO systems were investigated.

In this paper, we design the massive MIMO downlink, considering explicitly for path loss, Multi-User (MU). The similar model was analyzed in [6] for the uplink. We consider a large system limit where the number of BS antennas N and the number of UTs grow infinitely large at the same speed and derive approximations of achievable rates for different precoding strategies, i.e., Zero Forcing (ZF), Maximum Ratio Transmission (MRT/MRC), Regularized Zero Forcing (RZF) and our proposal Eigen Beam Forming Power Allocation (EBF-PA). These approximations are easy to compute and shown by simulations to be accurate for realistic system dimensions.

We further demonstrate that even a simple precoding scheme, such as EBF-PA, outperforms RZF by far and achieves a similar performance with one order of magnitude fewer antennas per UT. Our simulation results clearly explains proposed precoding scheme EBF-PA is well suited for large scale MIMO in LTE-A and MMB. Proposed system can outperform basic schemes in massive MIMO by using SINR of user and allocating the power at each UE.

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The paper is organized as follows: In Section II, we describe the system model and derive achievable DL rates of precoding schemes with the conventional techniques and proposed EBF-PA scheme. Section III contains power allocation scheme explaining the concepts with SINR and various achievable rate. In Section IV, we present some numerical results with LTE-A and MMB channel scenarios, in Section V we give the final conclusion based on our analysis of experimental results of precoding scheme in different channel environments. Additional mathematical algorithms are given in Appendix.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a cellular system consisting of BS equipped with N antennas and K (where, K<<N) UTs, each UT equipped with single antenna, as schematically shown in Fig. 2. Precoding of an antenna array is often said to direct the signal from the antenna array towards one or more receivers. The received signal acquires the perfect channel state information through pilot in TDD system. The composite received K×1 vector y at the users can be described as

\[
y = \sqrt{\rho}Hz + n
\]  

(1)

where, H is a composite K×N channel matrix, \(z\) is the transmitted vector across the N antennas, and \(n\) is a noise vector with unit variance. The M×1 transmit vector \(x\) contains a pre-coded version of the K×1 data symbol vector \(x\). Through pre-coding at the transmit side we have

\[
z = Gx
\]  

(2)

where, G is a NxK pre-coding matrix including power allocation to the data symbols. The vector x comprises data symbols from an alphabet \(\chi\) and each entry has unit average power, i.e., \(\mathbb{E}\{ |x_k|^2 \} = 1, k = 1, 2, 3, \ldots, K\). Taken together, the energy constraints on \(x\) and \(z\) yield power constraint on \(G: \text{Tr}(GH^H) = 1\), where \(\text{Tr} \ (\cdot)\) is the trace-operator and \((\cdot)^H\) denotes the Hermitian transpose.

A. Precoding Schemes

In this section, we discuss the conventional precoding schemes for large scale antenna system. We analyze ZF, MRT and RZF with our proposed EBF-PA scheme, whose performance is mainly based on the performance of power allocation scheme proposed in next section.

The channel matrix from the BS to K multiple UTs can be written as

\[
G = D^{1/2}H
\]  

(3)

Where, \(D = \text{diag}(d_1, d_2, \ldots, d_K)\), the component, \(d_k = \varphi \sqrt{\alpha_{sk}}\) consists of path loss and fading, \(\varphi\) is a constant related to carrier frequency and antennas gain, \(d_k\) is the distance between BS and UT k, \(\alpha\) is the path loss exponent, \(\zeta\) represents shadowing with the distribution of \(10 \\log_{10}(\zeta) \sim N(0, \sigma^2_{sh})\), where, \(\sigma^2_{sh}\) is the shadowing variance component and in fast fading channel matrix is \(H = [h_{1}, h_{2}, \ldots, h_{K}]^T\). Through fading channel, the received signal at all UTs can be expressed as, 

\[
y = Gx + n
\]  

(3)

And, the received signal for UT \(k\) can be written as

\[
y_k = \sqrt{\rho_k} h_k^H x + \sum_{j=1, j \neq k}^{K} \sqrt{\rho_j} h_j^H x_j + n
\]  

(4)

The received SINR for UT \(k\) can be given as:

\[
s_k = \frac{\rho_k}{\sum_{j=1, j \neq k}^{K} \sqrt{\rho_j} h_j^H h_j^H} + \sigma^2
\]  

and Spectrum efficiency for UT \(k\) can be given as

\[
S_k = \log_2 \left( 1 + \frac{\rho_k}{\sigma^2} \right)
\]  

(5)

\[
S_k = \log_2 \left( 1 + \frac{\rho_k}{\sigma^2} \right)
\]  

(6)

where, \(\mu = 2 \ln(5P_e)/3\) is the SNR gap between Shannon channel capacity and a practical scheme achieving the BER \(P_e\) [13]. Then, the total Spectrum Efficiency SE achieved by all UTs can be expressed as

\[
S_E(p) = \sum_{k=1}^{K} S_k = \sum_{k=1}^{K} \log_2 \left( 1 + \frac{\rho_k}{\sigma^2} \right)
\]  

(7)

\[
P = [P_1, P_2, \ldots, P_K]^T\] is the power allocation vector for all UTs at BS.

Now, since we have the expression for power allocation and SINR for the overall UE in the massive MIMO cell. Now, we just need to calculate the G matrix for each precoding schemes as Zero Forcing (ZF), Matched Filter (MF), Regularized Zero Forcing (RZF) and Eigen Beam Forming with Power Allocation (EBF-PA).

ZF Precoder

ZF pre-coding eliminates the interference by transmitting the signals towards the intended user with nulls in the “direction” of other users. The ZF pre-coder is given as.

\[
G_{ZF} = H^H
\]  

(8)

where, \(H^H = HH^H(HH^H)^{-1}\) is the pseudo inverse of the channel matrix H.

MF Precoder

The design of MF precoder based on CSI and using the combination of all CSI combined at BS. Where, H is the channel and \(c\) is the constant of power normalization.

\[
G_{MF} = cH^H
\]  

(9)

RZF Precoder

Here, the transmit weights are taken as, scalar power constraint, H is channel, I is channel coefficient [13].

\[
G_{RZF} = \hat{H}^H \left( \hat{H}^H \hat{H} + \Omega I \right)^{-1}
\]  

(10)

This constant \(\Omega\) may be optimized according to a variety of different metrics, many of which lead to convex optimization problems that require numerical solution. We use the result of [12] which minimized the sum of squared errors at all the user receivers, and obtained the simple, closed-form solution, as

\[
I = K\sigma_n^2
\]  

(11)

EBF-PA Precoder

Here, we made an assumption that the element of channel matrix H is Rayleigh distributed and the covariance matrix \(R_{HH} = E\{HH^H\}\) know by transmitter. We now diagonalise \(R_{HH}\) using Eigen Value Decomposition (EVD),

\[
R_{HH} = U_{HH} \Lambda_{HH} U_{HH}^H
\]  

(12)

where, \(D_{HH} = \text{Diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)\), \((\cdot)^H\) is the complex conjugate transpose operator. \(U_{HH}\) is unitary and \(\lambda_{ii}\) denotes \(i^{th}\) eigen value of \(R_{HH}\). It has been demonstrated that the
\( U_{up} = U_H^H \) beam direction satisfies. It uses dual channel estimate with power normalization for UE selection.

\[
\varrho_{\text{eff}} = \xi || H ||^2 \quad (\text{13})
\]

Finally, \( \xi \) is normalization scalar power constraint

### III. NOVEL POWER ALLOCATION ALGORITHM FOR LSAS

In this section, detailed analysis is given firstly, then the approximately optimal power allocation is given and the power allocation algorithm is developed.

According to the description in previous section, random channel vectors \( h_k \) are independent of random vector \( \mathbf{h}_k \). Denote \( \alpha_k \equiv \left[ \left| h_k \right|^2 \right]^2 / \left[ \left| h_k \right|^2 \right]^2 \), then \( \alpha_k \) is i.i.d Gaussian Random Variable (GRV) with parameters (1,1) according to [14]. Denote \( \alpha_0 \equiv \left[ \left| h_0 \right|^2 \right]^2 \), then \( \alpha_0 \) is also GRV with parameters (N,1). Since the expectation of \( \alpha_k \) is \( E[\alpha_k] = 1 \), then,

\[
E \left[ \left| h_0 \right|^2 \right] = \beta_k \sum_{i=1}^{K} \beta_i \left| h_i \right|^2
\]

and

\[
E \left[ \left| h_0 \right|^2 \right] = N \beta_k \beta_i
\]

As shown in (5), the SINR at UT k is complicated, and the optimal power allocation is impossible to be got by it. In order to develop the power allocation algorithm, we replace the denominator of \( \gamma_k \) with its expectation, i.e.,

\[
\gamma_k = \frac{\sum_{i=1}^{K} \beta_i \left| h_i \right|^2}{\sum_{i=1}^{K} \beta_i \left| h_i \right|^2 + \sigma^2}
\]

Especially, the condition \( K >> N \) is tough to meet considering large massive antennas array. Through the proper approximation by formula (16), the following theorem can be proved taking advantage of the convex optimization theory.

### IV. SIMULATION RESULTS

The simulation results are based on the link level Monte Carlo simulations. We have used two scenarios for our proposal simulation performance given LTE-A and another is MMB.

In the section-A, simulation parameters are based on 3GPP LTE-A 20 MHz bandwidth are given as

#### A. Simulations on LTE-A Environment

Table I shows the general simulation parameters and gives a definition of the environment simulated. Table II, shows the Power Delay Profile (PDP) of various channel scenarios in 3GPP LTE-A. In this case, we have performed the simulation for ETU case which has the maximum delay spread and simple case with Flat channel.

![Fig. 3. CDF performances of precoders in flat channel.](image)

In fig. 3, simulation result shows the performance of various CDF of precoding schemes in LTE-A Flat Channel. Since, in Flat channel the impulse response is stationary. So, we assume the ideal performances in this particular case. Here, the mean capacity \( E \left[ \log_2(1 + \text{SINR}) \right] \) is larger for the MF precoder compared to the ZF precoder. EBF-PA uses power allocation for all k number of users based on SINR of each user case., we can easily summarize the performance as lower to higher perfoamnce as ZF lower than MF, which performs lower than RZF and finally the EBF-PA which outperforms all the other schemes.
Fig. 4 simulation results show the performance of precoding schemes in ETU Channel. ETU channel performance is considered to be the worst channel performance. But, the performance is similar to that of the flat channel so we can say that the performance is of the schemes are unaffected by variation of channel impulse response. We can say that the EBF-PA can outperform the conventional algorithms in any scenario and can easily combat inter user interference and also the inter symbol interference.

Fig. 5, show the worst case scenario of the Fig. 5, but, we observe that the performance is not compromised in case of any precoding scheme. It proves the integrity of the system and secondly the performance of the EBF-PA, also shows the best amongst the conventional schemes.

B. Simulations on MMB-A/B/C Channel Environment

Table III shows the general simulation parameters and gives a definition of the environment simulated. Table IV, shows the power delay profile (PDP) of various channel scenarios in MMB –A/B/C channels. We can see the best channel case is MMB-A and worst case channel is MMB-C based on PDP. But, we discuss all the cases in MMB channel.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Frequency</td>
<td>28 GHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>500 MHz</td>
</tr>
<tr>
<td>Sample frequency</td>
<td>552.96 MHz</td>
</tr>
<tr>
<td>Subframe duration</td>
<td>1 ms</td>
</tr>
<tr>
<td>Subcarrier spacing</td>
<td>270 kHz</td>
</tr>
<tr>
<td>FFT size</td>
<td>2048</td>
</tr>
<tr>
<td>Occupied subcarriers</td>
<td>1728</td>
</tr>
<tr>
<td>No. of subcarriers/PRB</td>
<td>18</td>
</tr>
<tr>
<td>No. of available PRBs</td>
<td>96</td>
</tr>
<tr>
<td>CP size (samples)</td>
<td>512 (Extended CP)</td>
</tr>
<tr>
<td>No. of OFDM symbols/slot</td>
<td>27 (Extended CP)</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>QPSK, 16QAM</td>
</tr>
<tr>
<td>Noise</td>
<td>AWGN</td>
</tr>
<tr>
<td>Antenna Configuration</td>
<td>16x16</td>
</tr>
<tr>
<td>No. of Users</td>
<td>100</td>
</tr>
</tbody>
</table>

| Channel models             | MMB-A/AMMB-B/AMMB-C |

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\tau_{max}$ [ns]</th>
<th>$B_c$ [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMB-A</td>
<td>75</td>
<td>13.3</td>
</tr>
<tr>
<td>MMB-B</td>
<td>753.5</td>
<td>1.3</td>
</tr>
<tr>
<td>MMB-C</td>
<td>1388.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Fig. 7 shows the CDF performance of similar algorithms in MMB-A channel environment. Clearly, EBF-PA outperforms the conventional algorithms. But, if we compare the performance with LTE-A channel case the performance improves in this case.

Fig. 8 shows the CDF performance of in MMB-B channel environment. Here, as well EBF-PA outperforms the conventional algorithms, but, similar to previous case the performance improves in the case of MMB-B channel. So, if we implement EBF-PA algorithm in MMB channel we can analyze the performance of the basic algorithms.

Fig. 9 shows the CDF performance of in MMB-C channel environment. Compare to MMB-A/B channel the performance is decreased in this case due to increasing delay spread. But, also reduces for the other algorithms. In this case, the MMB-C system cannot prevent the loss in SINR of the system.

Fig. 10 shows the Spectrum Efficiency performance of proposed algorithms in MMB-A channel environment. The simulations of EBF-PA show very close performance to perfect CSIT case simulation proves it to be the best algorithm compared to conventional algorithms. Also, from LTE-A channel case the performance improves gradually for this particular case.

Fig. 11 shows the Spectrum Efficiency performance of proposed algorithms in MMB-B channel environment. The simulations of EBF-PA being very close to perfect CSIT case proves it to be the best algorithm compared to conventional algorithms. The performances in this case are very similar to the Fig. 10 case of MMB-A channel showing least effect in performance loss due to maximum delay spread.
Fig. 12 shows the loss in Spectrum Efficiency performance of proposed algorithms in MMB-C channel environment. Compare to other scenarios there is continuous loss in spectrum efficiency for all the mentioned conventional algorithms and also the proposed algorithm. So we can say that the performance of above mentioned algorithm deteriorates in MMB-C channel scenario or it fails to combat with the Inter Symbol Interference (ISI) and Inter User Interference (IUI) of LSAS System.

V. CONCLUSION

We show that the proposed precoding scheme EBF-PA is well suited for large scale MIMO environment with both environments LTE-A and MMB. Proposed system can outperform conventional schemes in massive MIMO by using SINR of user and allocating the power for each UE. Also, the simulation results prove that the performances of precoding schemes are very reliable for channel state information error reduction and better estimation of channel statistics at various Doppler spreads. We also observed that precoding scheme performances varies accordingly as per the channel environment (given for LTE-A and MMB) based on their varying maximum Doppler spreads. For the future prospect of research, we can implement proposed algorithm for estimating perfect and imperfect CSI cases. Moreover, they can be implemented for pilot contamination scenario for Uplink channel. Based on varying UT speeds, it can also be implemented with the time variant/ invariant channel cases.

APPENDIX

When the massive MIMO system adopts the precoding, the approximately power allocation vector achieving the optimal PA

\[ p^o = \left( \bar{H}^{-1} b \right)^{+} \]

where, \[ x^{+} = \max \{0, x\} \]

\[ b = [b_1, b_2, ..., b_K]^T, \]

and

\[ \bar{H} = \begin{bmatrix} \| b_1 \|_2^2 / \mu & 1 & \cdots & 1 \\ \vdots & \ddots & \ddots & \vdots \\ 1 & \cdots & \| b_K \|_2^2 / \mu \end{bmatrix} \]

where, \[ H = \text{diag}(\| b_1 \|_2^2, \| b_2 \|_2^2, ..., \| b_K \|_2^2) / \mu + I_{K \times K} \]

\[ b = [b_1, b_2, ..., b_K]^T \]

\[ b_k = \frac{\eta \| b_k \|_2^2}{\beta_k \ln 2} \]

and \[ H = \text{diag}(\| b_1 \|_2^2, \| b_2 \|_2^2, ..., \| b_K \|_2^2) / \mu + I_{K \times K} \]

\[ b = [b_1, b_2, ..., b_K]^T \]

\[ b_k = \frac{\eta \| b_k \|_2^2}{\beta_k \ln 2} \]

and \[ H = \text{diag}(\| b_1 \|_2^2, \| b_2 \|_2^2, ..., \| b_K \|_2^2) / \mu + I_{K \times K} \]

unique globally optimal power allocation. As the independence of \[ h_i (i = 1, 2, ..., K) \], the rank of the matrix \[ \bar{H} \] is \[ K \]. So , \[ \bar{H} \] is invertible and the power allocation expressed as \[ p^o = \left( \bar{H}^{-1} b \right)^{+} \]

REFERENCES