The Detection of the Linear Frequency-modulated Signal by Fractional Fourier Transform

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Abstract—The linear frequency-modulated signal (LFM) has been used widely in radar signals. In this paper, by using the characteristics of the linear frequency-modulated signal in fractional domain, three kinds of algorithms based on fractional domain have been put forward, that is, the maximum value detection, the energy concentration detection and the signal kurtosis detection. What’s more, the detection performance of each algorithm has been analyzed qualitatively.

Index Terms—Fractional Fourier Domain, LFM, signal detection

I. INTRODUCTION

The linear frequency-modulated signal (LFM) has been used widely in modern radar system. At present, in the electronic reconnaissance system, the detection and the identification of the linear frequency-modulated signal are often focused on the intrapulse analysis. The intrapulse analysis is distinguishing and identifying complicated modulated signal according to the minute characteristics of the intrapulse, in other words, by using time-frequency analysis, characterizing the signal in time-frequency domain to analyze the minute characteristics of the signal. In modern electronic confrontation signal environment, the number of the radiation sources becomes more and more; the modulation of the radiation source is very complicated; the signal sources are very crowded in the frequency domain, while dense and overlapped in the time domain. In such signal environment, time-frequency domain analysis needs complicated algorithm and a great deal of time. So the real-time detection and signal identification can not be achieved. In this paper, by using the characteristics of the linear frequency-modulated signal in fractional domain, the quick detection of the linear frequency-modulated signal in fractional Fourier domain has been accomplished, so achieving the aim of real-time signal detection.

II. FRACTIONAL FOURIER TRANSFORM

The Fourier transform operator, F, can be seen as the denotation of the signal on the time axis rotating anticlockwise around the origin by π/2 to the frequency axis. When the rotating angle of the operator R is an arbitrary angle α, the transform operator is called Fractional Rank Fourier Transform (FRFT). Based on the concept of rotation, the rotating operator R should have the characteristics as follows: rotate-zero \( R^0 = 1 \); consistency with normal Fourier transform \( R^2 = 1 \); addable rotation \( R^α R^β = R^{α+β} \); \( 2π \) periodic rotation \( R^{2π} = R^0 = 1 \). Fractional Fourier Transform will be explained in the following text.

\[
X_p(u) = R^α \{ x(t) \} = \int_{-∞}^{∞} K_p(t, u) x(t) dt
\]

\[
K_p(t, u) = \left\{ \begin{array}{ll}
\frac{1 - j \cot α}{2π} e^{j \left( \frac{Lu^2 + ut}{2} \right) \cot α - ut \csc α} & , α \neq nπ \\
\frac{2}{π} & , α = nπ
\end{array} \right.
\]

where \( p = \frac{π}{2} \) is called the rank of FFT, and let

\[
R^α = R^\frac{π}{2} = F^p . X_p(u)
\]

is called the p-th transform of \( X_p(u) \). Obviously, when \( p = 1 \), \( R^2 = 1 \) will degenerate into FFT. \( K_p(t, u) \) is called the core function of FRFT. FRFT has the same characteristics as common FFT, such as time-shift, frequency shift, scale, differential, multiplying, the proves of these characteristics and so on, which can be referred in paper [1~3]. In the following text, the characteristics of FRFT such as...
time shift, frequency shift and the FRFT transform of some signals will be explained:

time shift: \( R^\tau \{ x(t - \tau) \} \)
\[
= X_p(u - \tau \cos \alpha) e^{-j \frac{\tau^2}{2} \sin \alpha \cos \alpha - \tau \sin \alpha} \tag{2}
\]

frequency shift: \( R^\tau \{ x(t) e^{j \tau \omega} \} \)
\[
= X_p(u) e^{-j \frac{\tau^2}{2} \sin \alpha \cos \omega + \tau \sin \omega} \tag{3}
\]

FRFT of LFM signal: \( X_p(u) = R^\tau \{ e^{j\tau^2/2} \} \)
\[
= \frac{1 + j \tan \alpha}{1 + ct \tan \alpha} \left( \frac{2}{2 \tan \alpha} \right) e^{-j \frac{\tau^2}{2} \tan \alpha + \tau \cos \alpha} \tag{4}
\]

if \( \alpha = -\arctan \frac{\pi}{2} \) does not equal to an integral times of \( \pi \).

Take Chirp signal for example, \( x(t) = e^{j\omega_0 t + \frac{\omega_c}{2} t^2} \), do FRFT of the signal and let \( \alpha = \pi / 4 \), we can get:
\[
F_{\pi/4}[x(t)] = C \int_{-\infty}^{\infty} e^{-j \frac{\tau^2}{2} \tan \alpha + \tau \cos \alpha} dt
\]
\[
= Ce^{j\pi/12} \delta(\sqrt{2}u + f_0) \tag{5}
\]

Where C is the normalization constant, which is used to keep the energy invariable.

In fact, the function of Fractional Fourier is rotating the time-frequency structure of the to-be-analyzed signal by a definite angle, and for linear frequency modulated signal, by selecting an appropriate angle \( \alpha = \pi / 4 \), we can get an impulse signal, which aggregates its energy on its corresponding point on the u axis. However, if the selected angle does not match the signal, the original signal will be still transformed into a linear frequency modulated signal. So we can detect and identify LFM signal according to the concentration characteristic of LFM signal in fractional Fourier domain. In the following text, three kinds of algorithms based on the detection and identification of FFT are put forward.

III. THE MAXIMUM VALUE DETECTION METHOD IN FRACTIONAL FOURIER DOMAIN

Let the multi-component Chirp signal:
\[
x(t) = \sum e^{j(f_0 t + \frac{\omega_c}{2} t^2)} , \text{ according to formula (1-2), (1-3) and (1-4), we can get the FRFT as following:}
\]
\[
X_p(u) = \sum \frac{1 + j \tan \alpha}{1 + c \tan \alpha} e^{-j \frac{\tau^2}{2} \tan \alpha + \tau \cos \alpha} \tag{6}
\]

if \( \alpha = -\arctan \frac{\pi}{2} \) does not equal to an integral times of \( \pi \).

When \( \alpha = -\arctan c \), the square of the mode is:
\[
P_\alpha(u) = |X_p(u)|^2
\]
\[
= C \delta(u - (f_{00} \cos \alpha - \tau_0 \sin \alpha)) \tag{7}
\]

That is to say, when \( \alpha = -\arctan c \), the energy of the \( i \)th component will aggregate entirely on one point on the u axis, then engender the local maximum value on this point. Let p vary in the domain of [1,2] and bring forward the maximum value of every domain, then forming the maximum curve in fractional Fourier domain. The maximum value curve in fractional Fourier domain of a single LFM signal is shown in fig(1). The universe domain has a maximum value in its corresponding p domain, and suppose there is a gate detectable signal. The maximum value detection method can also be used to detect multi-component LFM signal. In fig(1), there are six maximum curves in fractional Fourier domain of six LFM signals, and six big pulse peak can be seen in the figure.

However, the maximum value detection method in fractional Fourier domain will be influenced greatly by noise, and it can not be used in the circumstance under very low signal-noise ratio. What’s more, the basic venation assumes the Gauss shape (shown as the broken line in fig(1), and self-adapted gate detection algorithm is so complicated that it is hardly successful to achieve, easily leading to visualalarm detection.

Fig(1) The maximum value detection of single LFM signal, SNR=+4dB, Gauss noise

Fig(2) The maximum value detection of multi-component signal, SNR=+3dB, Gauss noise

IV. THE ENERGY CONCENTRATION DETECTION

The energy concentration criteria of time-frequency distribution was firstly used by Baraniuk and Jones to complete the optimal core design. The high order ratio of the mode, \( L_p/L_q(p>q) \), which was put forward by Williams using R\( \acute {e} \)nyi information entropy and P.Flandrin, is used to evaluate the concentration characteristic of time-frequency distribution, and develops into an important concentration criteria. The basic idea is looking for an optimal window so that every point in the time-frequency domain can possess a minimum time-frequency support. Shown as fig(2), when the window completely matches the minimum time-frequency support, the energy is most aggregated. In fractional Fourier domain, for LFM signal, we can get a pulse signal easily by selecting an appropriate angle, which aggregates its energy on the
corresponding point on the u axis. Hereby the energy concentration on the fractional Fourier domain can be used to accomplish the detection.

\[ f \quad \text{Fig2(1) The minimum time-frequency support} \]

\[ t \]

\[ \text{Fig2(2) Energy concentration detection of single LFM signal, SNR=-5dB} \]

\[ \text{Fig2(3) Energy concentration detection of multi-component LFM signal, SNR=-3dB} \]

\[ M_n = \left| \int X_n(u) \frac{f}{\sqrt{du}} \right| \]

The high order ratio of the mode put forward by P.Flandrin is defined as:

\[ \Phi(\omega_1, \ldots, \omega_k) \]

\[ \text{is as followed:} \]

\[ \Phi(\omega_1, \ldots, \omega_k) \]

\[ \text{Accumulation can be defined as:} \]

\[ \text{cum}(x_1, \ldots, x_k) \]

\[ \text{And:} \]

\[ c_{4k}(\tau_1, \tau_2, \tau_3) = \text{cum}(x_1, \ldots, x_k) \]

\[ \text{Define the kurtosis is:} \]

\[ \varepsilon_x = \frac{m^4}{m^2} - 3 \]

The normalization kurtosis of Gauss signal is zero. If the kurtosis of the signal is negative, the signal is inferior Gauss signal; if the kurtosis of the signal is positive, the signal is super Gauss signal; it is obvious that pulse signal has big kurtosis. In each fractional domain, the linear frequency modulated signal varies from inferior Gauss signal to super Gauss signal. According to characteristic (2), the accumulation of multi-LFM signal can be divided into the sum of the accumulation of each component. When an angle in fractional domain matches one component(or its neighboring region), the component will change into super Gauss signal, and its kurtosis is big. If the other components can not be matched, it is still a LFM signal. If its kurtosis is positive, the kurtosis of the matched component will be much bigger; if negative, the variation in the scope of (-2, 0) will not greatly influence the kurtosis of the matched component. Therefore,
the kurtosis curve of the fractional domain assumes a rather big peak point. On the kurtosis curve of the fractional domain, multi-component LFM signal will assume many peak points, so the kurtosis detection algorithm can detect multi-LFM signal effectively.

The kurtosis curve of fractional domain based on four order accumulation is shown in fig 3. High order accumulation can completely inhibit any Gauss noise, so the kurtosis detection algorithm can be used under the circumstance of low signal-noise ratio, and then effectively inhibit every kind of Gauss noise. By using the normalized kurtosis, its basic venation assumes horizontal linearity, and the self-adapted detection algorithm is so simple that visual alarm will seldom appear.

VI. SIMULATION AND PERFORMANCE ANALYSIS

First, by adopting two different kinds of signals, single component LFM signal and multi-component LFM signal, under different signal-noise ratios, we have analyzed low signal-noise detection performance and multi-component detection ability of these three kinds of algorithms. It can be analyzed from table 1 that the maximum detection can be used to detect single component, multi-component LFM signal, but signal-noise ratio can not be too low; energy concentration detection has good detection ability under the circumstance of single component LFM signal and low signal-noise ratio, but does nothing to multi-component signal. However, local weighted algorithm can be used to complete the multi-component detection. The kurtosis detection has the detection ability of both single component and multi-component LFM signal, and also can complete the detection under the circumstance of low signal-noise ratio.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Signal</th>
<th>Single component</th>
<th>Multi-component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt;-4dB</td>
<td>&lt;-4dB</td>
<td>&gt;=3dB</td>
</tr>
<tr>
<td>The maximum value detection</td>
<td>Good</td>
<td>Worse</td>
<td>Good</td>
</tr>
<tr>
<td>energy concentration detection</td>
<td>Good</td>
<td>Good</td>
<td>Fail</td>
</tr>
<tr>
<td>The kurtosis detection</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
</tr>
</tbody>
</table>

VII. CONCLUSIONS

How to detect LFM signal quickly and effectively in the electronic reconnaissance system, especially under the circumstance of multi-component and complicated modulation, is a difficult problem all along. In this paper, we have completed quick detection of LFM signal in fractional domain, and put forward three kinds of algorithms based on fractional domain, maximum detection, energy concentration detection and kurtosis detection. What’s more, the detection performance of each algorithm has been analyzed qualitatively.

REFERENCES