Segmentation Based on Multifractal Region Variance of CCD X-Ray Cephalogram Lateral

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Abstract—Multifractal region variance approach used to estimate the multifractal spectrum is focused. The region variance probability based on X-ray image multifractal space is defined that can be used to estimate the multifractal spectrum. The multifractal spectrum property of the cephalogram is analyzed and the relationship between the multifractal spectrum and its weight factor is discussed, so the multifractal linear region of cephalogram is determined. The geometrical properties of its multifractal are verified. The segmentation of the cephalogram is detected. The above approach shows that the noise can be repressed effectively, the local property of an image can be remained as much as possible and the fine property can be stressed. The valuable edges of the cephalogram are clearer and more correct.

Index Terms—fractal, segmentation, multifractal, X-ray, cephalometric

I. INTRODUCTION

CEPHALOGRAM analysis is a basic method to diagnose and treat patients with facial skeleton and dental abnormalities in the fields of orthodontics and orthognathia surgery[1]. It has experienced three steps of manual, computer assisted and computer auto-recognition. The first two methods depend on human to recognize the Landmarks of Cephalogram tediously and imprecisely[2], [3]. And computer auto-recognition consists of two steps, ① edge detection of soft and bone tissues in X-ray cephalograms, ② Landmarks recognition automatically. Landmarks are some of characteristics related to the edges of facial anatomy as shown in Fig.1. Automatic cephalometric analysis has been a subject of research for many years and the automatic location of landmarks has been attempted by more than 20 independent researchers with varying degrees of success. The various methods used by researchers in this field can be divided into three categories. The first one employs knowledge-based edge tracker to reduce the search area, followed by imaging-matching techniques to pin point the exact location of the landmarks. The second category employs artificial intelligence and neural networks.

The disadvantage of first category is that the success of the approach is very much dependent on the quality of the X-ray and can only locate a landmark if it is positioned on an edge. The second method only works if images are registered in a general space, having the same sizes, rotations and shift. A small shift in the head in the X-ray will reflect a large shift in the locations of landmarks.

Fig.1. Most commonly landmarks

The third category is to extract the edges of soft and bone tissues in the cephalograms and then to locate the landmarks. Because most of landmarks are related to the edges of soft and bone tissues in the cephalograms, edge detection is the basic and important step to recognize landmarks. Thus, it is very important that the edges of soft-tissues and bone-tissues are extracted correctly for more precise Landmarks.

The principle of X-ray imaging is different from the other imaging because the X-ray image is obtained by absorbing X-ray. The different objects or different parts of the same object have the different absorbance and the absorbed amount of X-ray is the overlay of each of them. So the X-ray image has high overlapping, high noise, low contrast, much amount of data, high resolution needed. We have to say it is very difficult to process X-ray image and the classical methods are faced a difficult challenge, especially for X-ray cephalogram. Cephalogram analysis is still at the beginning of the study. It is very difficult to precisely extract the tissues edge in the cephalogram laterals because of its complicated structure that soft tissues and bone tissues are overlaid each other. Another reason is that the quality of the X-ray cephalogram will influence the result of edge detection. Laplacian and Canny algorithms or knowledge-based contour search technology were used to extract the edges of soft and bone tissues. But no good results were obtained [4]-[7]. Until now, no clear, continuous outline is obtained.
Natural images have been proved that can be described by fractal, and most of outlines of the cephalogram laterals have been extracted clearly by using Fractional Brownian Motion model [8]-[10]. But the complex overlaps can not be detected expectably, e.g. pterygomaxillary fissure, et al.

Cephalogram images can be described with irregular multifractal in the appropriate scale. The multifractal spectrum can all-out describe the characters of distribution for the different geometric structure or different physical property[11],[12]. In fact, the edge of an image can be defined with probability distribution in the given scale as well as its geometric characters. Using multifractal to extract the edge has the advantages. The approach based on grey gradient operation only considers the geometric character of the edge. The multifractal method considers the statistical characters by multifractal spectrum \( f(\alpha) \) as well as the geometric characters by H Õlder odd exponent \( \alpha \). So that the main edge information can be remained and stressed as well as the minor edge information can be neglected.

So far, several algorithms have been proposed for the computation of multifractal parameters. In remote sensing, multifractal analysis has been successfully applied in several applications such as the analysis and classification of land [13] and sea-ice [14]. One of the commonly used methods was presented by Chaudhuri and Sarkar [15]. The method was based on the differential box-counting (DBC) algorithm and has been shown to produce fairly good results. In medicine, the separation of the cell on its core and its cytoplasm by the use a multifractal algorithm based on the computation of the singularity exponent on each points of the image had been achieved[16].Different methods for determining the probability will effect the multifractal spectrum. Four methods [A. absolute deviation from average height, B. variance from average height, C. height based on the minimum height of rough surface and D. height based on the bottom interface of the thin film] are considered in detail at the multifractal analysis [17].

In this paper, the probability based on X-ray image multifractal space is defined that can be used to estimate the multifractal spectrum. The region variance probability based on X-ray image multifractal space is defined that can be used to estimate the multifractal spectrum. The multifractal linear region of the cephalogram and multifractal spectrum are analyzed. The geometrical properties of its multifractal are verified. The edge of cephalogram is detected. It is indicated that the edges of inner bone tissues of cephalogram can be detected more clearly with multifractal region variance approach introduced in this paper than Sobel and Canny.

II. MULTIFRACTAL AND ITS SPECTRUM

Mandelbrot observed the multi-scale property first when he studied the turbulence in 1974 and he used the word non-lacunary to describe the phenomena in his book *The fractal geometry of Nature*. Grassberger, Hentschel and Procaccia developed the concept when they researched the odd attractor and gave the general formula.

We divide the studied object \( F \) into \( N \) different regions \( s_i \) \( (i=1,2,3⋯N) \), let \( \varepsilon \) is the size of the \( i_{th} \) region and \( p_i \) is the generated probability of this region, the different region \( s_i \) corresponding different \( p_i \).

The sets composed by all probability \( p_i(\varepsilon) \) are divided into the serial subsets, i.e. according to the size of \( p_i(\varepsilon) \), to construct the following subsets, satisfied with the power function

\[
p_i(\varepsilon) \propto \varepsilon^{\alpha_i}
\]

Here \( \alpha_i \) is called odd exponent, its value is related to the corresponding subset. If the fractal measurement is even (e.g. here mass), the value of \( \alpha_i \) must be only one value. If it is not even, we can use the size of \( \alpha_i \) value to distinguish the different subsets. The value of \( \alpha_i \) reflects the size of the generated probability corresponding this region.

We define that the relationship between the number of unit \( N(\varepsilon) \) in the subset and the size \( \varepsilon \) is

\[
N(\varepsilon) \propto \varepsilon^{-f(\alpha)} \quad (\varepsilon \rightarrow 0)
\]

Compared with the definition of the fractal dimension, \( f(\alpha) \) indicates the fractal dimension of the subset for the same \( \alpha_i \) value. \( f(\alpha) \) is called multifractal spectrum.

Define a partition function \( \chi_q(\varepsilon) \)

\[
\chi_q(\varepsilon) \equiv \sum p_i(\varepsilon)^q = \varepsilon^{\alpha(q)} \quad (2-3)
\]

If the latter equation is satisfied, i.e. the partition function \( \chi_q(\varepsilon) \) and the size \( \varepsilon \) satisfy the power function, then we can get the following result from the slope of the logarithm, \( \ln \chi_q - \ln \varepsilon \)

\[
\tau(q) = \frac{\ln \chi_q(\varepsilon)}{\ln \varepsilon} \quad (\varepsilon \rightarrow 0)
\]

here \( \tau(q) \) is called mass exponent.

When the system satisfies multifractal, according to the (2-3), we can obtain

\[
\tau(q) = a q - f(\alpha) \quad (2-5)
\]

\[
\alpha = \frac{d \tau(q)}{dq} \quad (2-6)
\]

It is very important to calculate the probability measurement when analyzing the multifractal spectrum. The experiment shows that the different dividing fractal set \( F \), different calculating steps and the different calculating method will get a set of \( f(\alpha) \) that has the different geometrical property with a certain correlation.

Define the generalized fractal dimension is

\[
D_q = \frac{\tau(q)}{q-1} = \frac{\ln \chi_q(\varepsilon)}{(q-1)\ln \varepsilon} \quad (\varepsilon \rightarrow 0)
\]

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III. THE REGION VARIANCE PROBABILITY BASED ON X-RAY IMAGE MULTIFRACTAL SPACE

Let $B(x, y)$ is a $M \times M$ gray image. Consider the region of $\varepsilon \times \varepsilon$. $I(i, j)$ is the pixel of the point $B(i, j)$. Consider the $3 \times 3$ window of the point $B(i, j)$ shown in Fig.2.

$$I_k(i, j) = \frac{1}{5}[B(i, j) + B(i-1, j) + B(i+1, j) + B(i, j+1) + B(i, j-1)]$$

Then we can define the normalization multifractal region variance probability of $k^{th}$ window is

$$p_k(\varepsilon) = \frac{|I_k(i, j) - \overline{I}_{av}(i, j)|^2}{\sum |I_k(i, j) - \overline{I}_{av}(i, j)|^2}$$

(3-1)

where $\overline{I}_{av}(i, j)$ is the mean of all $I_k(i, j)$.

Now the probability of $k^{th}$ window can be mapped into the probability of point $B(i, j)$, and let

$$p_k(i, j) = p_k(\varepsilon) = \frac{|I_k(i, j) - \overline{I}_{av}(i, j)|^2}{\sum |I_k(i, j) - \overline{I}_{av}(i, j)|^2}$$

(3-2)

$p_k(i, j)$ is called the region variance probability of point $B(i, j)$.

If the probability of region $m$ and $j$ are $p_m$ and $p_j$, and $p_m >> p_j$, when $q >> 1$, $p_m^q$ in (3-2) will contribute the main effect. The experiment shows that the multifractal spectrum $f(\alpha)$ will be steady if only weight factor $q$ is limited at a certain range.

IV. ANALYZING MULTIFRACTAL OF CEPHALOGRAM

The sample image is a $512 \times 512$ CCD cephalogram lateral shown in Fig.3.

A. Analyzing the Linear Region of Cephalogram Multifractal

Analyzing the linear region of the multifractal spectrum is to study the linear relationship between the logarithm of the partition function $\chi_q(\varepsilon)$ and the logarithm of the size of the cephalogram. That is to say, if the plot of $\ln \chi_q - \ln \varepsilon$ appears approximate straight line, we can say that it has the expectable linear property.

Suppose $B(x, y)$ is a grey image and $B(i, j)$ is its pixel at the point $(i, j)$.

1) Calculate the probability measurement

The region variance probability of point $B(i, j)$, $p_v(\varepsilon)$ is calculated by formula (3-2).

(2) Draw the plot of $\ln \chi_q - \ln \varepsilon$

According to the formula (2-3), Let the weight factor $q = -50 \sim +50$, calculate the logarithm of the partition function $\chi_q(\varepsilon)$, $\ln \chi_q$. Then draw the plot of $\ln \chi_q - \ln \varepsilon$, shown in Fig.4. It shows that the plot $\ln \chi_q - \ln \varepsilon$ has the expectable linear property.

B. Verifying and Analyzing the Geometry of the Multifractal Spectrum

Calculate the mass exponent $\tau(q)$, odd exponent $\alpha(q)$ and multifractal spectrum $f(\alpha)$ each other according to the formula (2-4), (2-5) and (2-6) and draw the plots. We obtain the following results:

The geometry of the cephalogram shown in Fig.3 shows that $\alpha(q)$ is a strict monotone decreasing function about $q$ in the plot $q - \alpha$ shown in Fig.5, mass exponent $\tau(q)$ is a strict
monotone increasing convex function about $q$ shown in Fig.6 and the multifractal spectrum $f(\alpha)$ is convex about $\alpha$ and $f(\alpha)$ has its maximum value 2 when $\alpha = 0$ shown in Fig.7. This is because that each pixel of the image has some probability distribution, i.e. $p(i,j) \neq 0$. That plot of $\alpha - f(\alpha)$ appears hook shape shows the unit numbers of maximum probability subset are greater than the unit numbers of minimum probability subset.

V. EDGE DETECTING OF CEPHALOGRAM

The steps of the edge detection based on the multifractal are

A. Choose the adaptive scale $\varepsilon$ (here $\varepsilon = 3$), then the window size is $\varepsilon \times \varepsilon$;

B. Calculate the probability $p_s(i,j)$ of the point of $B(i,j)$ according to the formula (3-2);

C. Repeat the step A) and B) for different windows;

D. Calculate the partition function $\chi_q(\varepsilon)$ and its logarithm according to the formula (2-3);

E. Calculate the odd exponent $\alpha_i$, the mass exponent $\tau(q)$ and the multifractal spectrum $f(\alpha)$ according to the formula (2-6), (2-4) and (2-5);

F. Choose an adaptive threshold of $f(\alpha)$, then obtain the edge of the cephalograms, shown in the Fig.8.

The results of edge detection with Sobel and Canny are shown in Fig.9 and Fig.10. It is obvious that much important edge information is lost in the Fig.9 and Fig.10, such as Porion, Sella and Orbitale, and some of non-edge information is treated as edges such as pterygomaxillary fissure in the Fig.10. And Orbitale, Anterior Nasal Spine is confused in Fig.9. So the multifractal variance approach is better to segment the bone tissue of the cephalogram than Sobel and Canny.

Comparing with the traditional methods, such as absolute deviation from average height, variance from average height, height based on the minimum height of rough surface and height based on the bottom interface of the thin film, the algorithm introduced above can denoises better and the weight factor $q$ can converge quickly. The plot of weight factor $q$ and the multifractal spectrum $f(\alpha)$ for the method of bottom interface of the thin film is shown in fig.11 and the processing result for the image of fig.3 is shown in fig.12. It is indicated that the segmentation result has been interfered by noise, so the bone tissue segmentation is not ideal.
The above discussion shows that the CCD cephalogram lateral has multifractal characteristics and the edge detection can use multifractal approach. Multifractal region variance approach used to estimate the multifractal spectrum shows that the noise can be repressed effectively, the local property of an image can be remained as much as possible and the fine property can be stressed. The result of edge detecting shows that the bone tissues of cephalogram are much clearer than the current methods.

REFERENCE

